Selection Of Terminal Sires And Dams For Meat Producing Animals Sold Under A Grid Pricing System.

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Introduction.

Meat animals are often priced per unit of carcass weight based on a price grid, which depends on the category of one or more traits. In pig production, the price per kilogram depends on both weight class and lean yield or carcass quality class. Similar grids are used for pricing beef cattle and fish. This type of pricing differs from that described by Rohr *et al.* (1999) in which trait categories determine the total carcass price, not the price per kilogram.

Producers raise stock to the optimal market age for their herd and select parents on weight, meat quality, etc. at fixed age, or feed stock to the optimal weight or quality and select parents on age, etc. at fixed weight or quality (Wilton and Goddard (1996)). Different management practices may be required to maximize net revenue from alternative grids, e.g. producers selling stock at fixed age may need to sell at a lighter weight or younger age under a different price grid. The optimum strategy for each producer should be chosen based on the expected net revenue at the optimum management for each grid.
Initially, we assume days at market age, (age_m), is fixed at optimum. Net profit generated by a terminal sire or dam is:

$$NP = no\left(\mathrm{R}_{\mathrm{Grid}} \times cwt - age_m \mathrm{days} \times \mathrm{costs/day}\right)$$

where *no* is the number of market offspring, R_{grid} is the expected price paid per market animal, *cwt* is the expected offspring carcass weight and costs/day include feed and service costs. Note that despite the fact that the grid prices are discrete, *NP* is continuous in the expected offspring trait values. Genetic contributions to market animal traits depend on the parent breed or line averages, w_A and w_B, their breeding values, g_{sire} and g_{sdam} and any heterosis between the parent breeds, h_{AB}:

$$\mu_g = (\bar{w}_A + \bar{w}_B + g_{sire} + g_{dam})/2 + h_{AB}$$

Parent economic values are calculated setting either g_{dam} or g_{sire} to the herd average, or zero; the factor of ½ is not necessary for ranking, and is ignored in the rest of this paper.

Methods

Grids in which the price/kg depends on trait(s) other than weight. As an example of this type of price grid, suppose the class of a trait other than *cwt*, e.g. muscle depth, with *m* classes defined by upper limits c_1 to c_{m-1} and > c_{m-1}, determines the per *kg* price of an animal. If the prices/class are given by Q_{1xm}, expected offspring muscle depth is μ_{md} and offspring

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variance is $\sigma_{md}^2 = \sigma_{md\,p}^2\ (1-h_{md}^2/4)$, where h^2 is the heritability and $\sigma_{md\,p}^2$ the herd variance of muscle depth, the expected revenue from the sale of one offspring is $R_{Grid} = Q_{1xm}T_{mx1} \times cwt$, where T is the probability muscle depth is in each class,

$$T' = \left(\int_{-\infty}^{c_1} \phi(x:\mu_{md},\sigma_{md})dx \quad \dots \quad \int_{c_{i-1}}^{c_i} \phi(x:\mu_{md},\sigma_{md})dx \quad \dots \quad \int_{c_{m-1}}^{\infty} \phi(x:\mu_{md},\sigma_{md})dx \right)$$

and $\phi(x:\mu_{md},\sigma_{md})$ is the normal density function. Since

$$\frac{\partial}{\partial\mu}\int_a^b \phi(x:\mu,\sigma)dx = \int_a^b \frac{\partial}{\partial\mu}\phi(x:\mu,\sigma)dx = \int_a^b -\frac{\partial}{\partial x}\phi(x:\mu,\sigma)dx = \phi(a:\mu,\sigma) - \phi(b:\mu,\sigma)$$

the economic value for muscle depth is:

$$\frac{\partial NP}{\partial\mu_{md}} = no\left(\frac{\partial \mathrm{R_{Grid}}}{\partial\mu_{md}} \times cwt\right) = no\left(QU \times cwt\right),$$ where

$$U' = \left(-\phi(c_1:\mu_{md},\sigma_{md}) \quad \cdots \quad \phi(c_{i-1}:\mu_{md},\sigma_{md}) - \phi(c_i:\mu_{md},\sigma_{md}) \quad \cdots \quad \phi(c_{m-1}:\mu_{md},\sigma_{md})\right)$$

and that for carcass weight is $\partial NP / \partial cwt = no\left(R_{Grid}\right) = no\left(QT\right)$. Both expressions are evaluated at the current commercial herd means and variances.

If grid prices also depend on classes defined by an additional trait, e.g. lean yield, with n classes defined by upper limits d_1 to d_{n-1} and > d_{n-1}, offspring mean, μ_{ly}, and variance σ_{ly}^2, the expected revenue per offspring is $R_{grid} = S'_{1\times n}Q_{n\times m}T_{m\times 1} \times cwt$, where $Q_{n\times m}$ is now an mxn matrix of prices,

$$\mathrm{S}' = \left(\int_{-\infty}^{d_1} \phi(y:\mu_y,\sigma_y)dy \quad \cdots \quad \int_{d_{j-1}}^{d_j} \phi(y:\mu_y,\sigma_y)dy \quad \cdots \quad \int_{d_n}^{\infty} \phi(y:\mu_y,\sigma_y)dy \right)$$

and T is as previously defined. The economic value for lean yield is $no \times V'QT \times cwt$ where

$$V' = \left(-\phi(d_1:\mu_{ly},\sigma_{ly}) \quad \cdots \quad \phi(d_{i-1}:\mu_{ly},\sigma_{ly}) - \phi(d_i:\mu_{ly},\sigma_{ly}) \quad \cdots \quad \phi(d_{n-1}:\mu_{ly},\sigma_{ly})\right)$$

and that for muscle depth is $no \times U'QS \times cwt$, where U is as previously defined, both evaluated at the current commercial herd means and variances.

Grids in which the price/kg depends on the carcass weight category. For this type of grid, the carcass weight class of the animal determines the per kilogram price. Suppose the grid depends on k weight classes defined by upper limits e_1 to e_{k-1} and > e_{k-1} with per kg prices given by Q_{1xk}, the offspring mean and variance of cwt is μ_w and σ_w^2.
The expected revenue per animal is: $R_{grid} = Q_{1\times k}W_{k\times 1}$, where

$$\mathrm{W}' = \left(\int_{-\infty}^{e_1} x\phi(x:\mu_w,\sigma_w)dx \quad \cdots \quad \int_{e_{i-1}}^{e_i} x\phi(x:\mu_w,\sigma_w)dx \quad \cdots \quad \int_{e_{n-1}}^{\infty} x\phi(x:\mu_w,\sigma_w)dx \right)$$

$$= \left(-\sigma_w\phi(e_1) + \mu_w\Phi(e_1) \quad \cdots \quad \sigma_w(\phi(e_{i-1}) - \phi(e_i)) + \mu_w(\Phi(e_i) - \Phi(e_{i-1})) \quad \cdots \quad \sigma_w\phi(e_{k-1}) + \mu_w(1 - \Phi(e_{k-1}))\right)$$

is the average weight per class times the probability the animal is in that class, $\phi(e_i)$ is the standard normal density function evaluated at $(e_i-\mu_w)/\sigma_w$, and $\Phi(e_i)$ is the cumulative standard normal distribution evaluated at $(e_i-\mu_w)/\sigma_w$.

Since $\frac{\partial\phi(x:\mu,\sigma)}{\partial x} = -\frac{\partial\phi(x:\mu,\sigma)}{\partial\mu}$, exchanging the order of integration and differentiation, and integrating by parts, the economic value for *cwt* is:

$$\frac{\partial NP}{\partial\mu_w} = no\frac{\partial R_{grid}}{\partial\mu_w} = no(\mathrm{QZ}),\quad \mathrm{Z} = \begin{pmatrix} -e_1\phi(e_1:\mu_w,\sigma_w) + \Phi(e_1) \\ \vdots \\ -e_i\phi(e_i:\mu_w,\sigma_w) + \Phi(e_i) + e_{i-1}\phi(e_{i-1}:\mu_w,\sigma_w) - \Phi(e_{i-1}) \\ \vdots \\ e_{k-1}\phi(e_{k-1}:\mu_w,\sigma_w) - \Phi(e_{k-1}) + 1 \end{pmatrix}$$

evaluated at the current commercial herd mean and variance.

Carcass prices are for pigs in Canada are based on a two-trait grading grid, in which per kilogram prices depend on both *k* weight classes and *n* lean yield classes. The revenue per pig is $R_{grid} = S'_{1\times n}Q_{n\times k}W_{k\times 1}$,where *W* and *S* are as previously defined, and $Q=(q_{ij})$ represents the price in weight class *j* and lean yield class *i*. The economic value for carcass weight is $\partial R_{grid}/\partial\mu_w = no\times S'QZ$ and that for lean yield is $\partial R_{grid}/\partial\mu_{ly} = no\times V'QW$ where Z and V are as previously defined. Both expressions are evaluated at current herd means and variances.
The example below shows this type of grid applied to commercial pig prices.

Example: Canadian pig prices: 2 trait grid. Prices per kilogram of carcass weight for Canadian market pigs are determined by grids with classes defined by weight and lean yield ranges. Suppose a producer sells pigs at the optimum age of 162 days, with average carcass weight and standard deviation 92±4*kg*, average lean yield 60±1%, and prices per *kg* for weight ranges <70*kg*, 70-85*kg*, 85-100*kg*, and >100*kg* and lean yield ranges <55%, 55-61% and >61% given by the columns and rows respectively of the matrix *Q* times $1.25 base price. If the heritability of carcass weight is 35%, and of lean yield is 45%, the expected standard deviations of offspring carcass weight and lean yield from a single parent are 3.82*kg* and 0.94%, respectively. The price grid, Q, and S,W,Z and V for this example are:

$$Q = \begin{pmatrix} .5 & .85 & .90 & .85 \\ .5 & .90 & 1.05 & .95 \\ .5 & .95 & 1.1 & 1.0 \end{pmatrix}, S = \begin{pmatrix} .00 \\ .86 \\ .14 \end{pmatrix}, W = \begin{pmatrix} 0.00 \\ 3.01 \\ 88.96 \\ 0.03 \end{pmatrix}, Z = \begin{pmatrix} -.00 \\ -1.62 \\ 2.59 \\ 0.03 \end{pmatrix}, V = \begin{pmatrix} -.00 \\ -.24 \\ 0.24 \end{pmatrix}$$

The expected revenue per pig sold is $S'QW$=$121; the economic value for lean yield is $no\times V'QW = no\times$ $1.11/% and that for carcass weight is: $no\times S'QZ = no\times$ $1.30/*kg.*

Equivalently, the producer could select parents on predicted offspring growth rate (*gr)* to market age instead of weight at market age. Assuming growth per *kg* per day is linear over the range of interest, e.g. $cwt=gr\times days$, the economic value for growth rate is equal to that value for *cwt* times 162 days market age = $no\times$ $210/kg/day.

Alternative parameterization: economic values for age at fixed weight. Many pig producers feed market animals to a fixed weight, cwt_m, rather than a fixed age, and need an economic value for age at market weight. Since offspring net profit is continuous, the optimum market weight is equal to the herd weight at the optimum age under the previous parameterization, and we can reparameterize *NP* in terms of carcass weight instead of age. The economic weights will be equivalent (Wilton and Goddard (1996); Quinton and Wilton (2008)). For example, if age=cwt_m/gr, the revenue now depends on the offspring mean and variance of market age, $NP=no(R_{grid}- cwt_m \times \text{costs}/kg)$. The economic value for age at cwt_m is calculated as a transformed trait, from that for growth rate by dividing by the rate of change of age with respect to growth rate at the current herd mean (Quinton, Wilton, Robinson *et al*. (2006)); *i.e.*

$$\frac{d(NP)}{dgr} \div \frac{d(age)}{dgr} = no \times \frac{210.17}{(-cwt_m) \div gr^2} = -no \times \frac{210.17}{92} gr^2$$

The current herd mean growth rate is gr=92÷162=0.57kg/day, so the economic value for age at market weight is -$no\times$ \$0.74/day.

Discussion.

Most price grids change every few years. Producers selecting parents at the multiplier level should anticipate any such changes before the market animals bred from their stock are sold. Sire and dam line breeders will need to predict such changes in all the grids used by their buyers much further into the future.

In general, the economic values depend on the price grid chosen, the optimum market age or weight, and the phenotypic means and variances of the commercial animals, and can differ considerably over the normal range of commercial herds. The calculations shown above are quite straightforward and can be performed using R (2005), or many commercially available spreadsheet packages.

References

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